STRUCTURE CALCULATION OF $^{16}_{\eta}O$ NUCLEUS

Hla Hla Win[1], Khin Nilar Myo Thet[2], Theingi[3]

Abstract

The purpose of this research is to investigate the structure of η-mesic oxygen nucleus theoretically which is a bound system of η-meson and Oxygen core nucleus. Since original functional form of η-N interaction cannot be analytically solved, we have transformed the functional form into Gaussian form by applying the Gauss elimination method. The transformed η-N potentials are equivalent to that of original potential by using the optimum sets, $\mu_1 = 0.05$ fm, $\mu_N = 20$ fm and N=20. The η-nucleus interaction is obtained by folding the η-N interaction with Oxygen nucleus nuclear density. By applying the η-nucleus folding potential we have computed the binding energy of $^{16}_{\eta}O$ system which is 3.2814 MeV and its level width is 0.3704 MeV.

Keywords:, Gauss elimination method, folding potential, Power Inverse Iteration Method, resonance energy

Introduction

The existence of η-mesic nucleus was first predicted by Q. Haider and L.C. Liu in 1986. It is a consequence of the attractive interaction between the η meson and all the nucleons in the nucleus. The attractive nature of the interaction follows from the work of R.S. Bhalerao and L.C. Liu who found, from a detailed coupled-channel analysis of πN → πN, πN → ππN, and πN →ηN reactions, that near-threshold ηN interaction is attractive. The reaction mechanisms used for eta production are usually based on models similar to those used for other mesons such as the pions and kaons.

Since its discovery, extensive theoretical and experimental efforts have been devoted towards achieving a better understanding of the η-meson properties and its interaction with other particles. This was due to the special role played by the η-meson in particle physics. The latter can be attributed to quantum mixing of the quark states corresponding to the η and π^0 mesons.

Although the η-meson is four times heavier, it is in many respects similar to the π^0-meson. Both are neutral, spinless and have almost the same lifetime, $\sim 10^{-18}$ s. The kindship between the two mesons manifests itself very clearly in their decay modes. They are the only mesons that have a high probability of pure radiative decay. The pion almost entirely (98.798%) decays into the radiative channel $\pi^0 \to \gamma + \gamma$. For the η the purely radiativedecay is also the most probable mode,

1. Lecturer, Department of Physics, Taunggyi University
2. Master of Research student, Department of Physics, University of Mandalay
3. Associate Professor, Department of Physics, University of Distance Education

$$\eta \rightarrow \begin{cases} \gamma + \gamma & (38.8\%) \\ \pi^0 + \pi^0 + \pi^0 & (31.9\%) \\ \pi^+ + \pi^- + \pi^0 & (23.6\%) \\ \pi^+ + \pi^- + \gamma & (4.9\%) \\ \text{otherdecays} & (0.8\%). \end{cases}$$

Therefore, when π^0 and η are viewed as elementary particles, they look quite similar. However their interaction with nucleons is different. The experimental searches involve the production of η mesons and hence signals for the existence of eta-mesic states via their possible decay modes and final state interactions of eta mesons with nuclei.

Resonances

The first resonance in particle physics was discovered by H. Anderson, E. Fermi, E. A. Long, and D. E. Nagle, working at the Chicago Cyclotron in 1952. Resonance states are formed when quantum particles collide at certain (resonant) energies. Before moving apart, they stay together for a while. During the resonance lifetime, the particles move around each other and "forget" the direction from which they came. Therefore, when the resonance eventually decays, the particles "choose" the direction to move away at random.

A resonance can be viewed and approached from two different angles, as a delay connected with an enhanced phased shift in a scattering process or as a long-lived but decaying state of a compound system. The main observable characteristics of a resonance are position and the width. The real and imaginary parts of the energy give the position and width of the resonance, respectively.

A resonance energy is $E_{res} = E_r - i\frac{\Gamma}{2}$ has a negative imaginary part , which is called resonance width. The use of a complex energy allows a classification of the energy levels of a quantum system.

Two-Body Calculation

We use the Gaussian basis wave function as the total wave function of our two-body system which has the following form;

$$u(r) = r^{\ell+1} \sum_j c_j e^{-\left(\frac{r}{b_j}\right)^2} \qquad (1)$$

where c_j's expansion coefficient and b_j's are range parameters.

The Schrödinger equation is written as

$$H\Psi = E\Psi \qquad (2)$$

The two body Hamiltonian is

$$H = -\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2} + V(r)$$

where reduce mass , $\mu = \frac{m_1 m_2}{m_1 + m_2}$

And then we can write as;

$$\sum_{i,j} [T_{ij} + V^{\ell}_{ij} + V_{ij}] c_j = E \sum_{i,j} N_{ij} c_j \tag{3}$$

$$\sum_{j=1}^{N} H_{ij} c_j = E \sum_{j=1}^{N} N_{ij} c_j \tag{4}$$

Eq.(4) can be written as

$$H_{11} C_1 + H_{12} C_2 + ... + H_{1N} C_N = E(N_{11} C_1 + N_{12} C_2 + ... + N_{1N} C_N)$$
$$H_{21} C_1 + H_{22} C_2 + ... + H_{2N} C_N = E(N_{21} C_1 + N_{22} C_2 + ... + N_{2N} C_N)$$
$$.$$
$$H_{N1} C_1 + H_{N2} C_2 + ... + H_{NN} C_N = E(N_{N1} C_1 + N_{N2} C_2 + ... + N_{NN} C_N)$$

The above N equations can be written as a matrix form as follows;

$$\begin{bmatrix} H_{11} & H_{12} \cdots & H_{1N} \\ H_{21} & H_{22} \cdots & H_{2N} \\ & & \\ H_{N1} & H_{N2} \cdots & H_{NN} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \\ C_N \end{bmatrix} = E \begin{bmatrix} N_{11} & N_{12} \cdots & N_{1N} \\ N_{21} & N_{22} \cdots & N_{2N} \\ & & \\ N_{N1} & N_{N2} \cdots & N_{NN} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \\ C_N \end{bmatrix} \tag{5}$$

$$[H][C] = E[N][C] \tag{6}$$

$$[N]^{-1}[H][C] = E[C] \tag{7}$$

$$[A][C] = E[C] \tag{8}$$

$N_{ij}, T_{ij}, V^{\ell}_{ij}$ are analytically solved by using standard integral. We solved the two body Schrödinger equation to obtain the resonance energies. By diagonalization the Hamiltonian matrix elements, we obtained the complex energy eigenvalues.

Eta-Nucleon Interaction

In our calculation, we use the potential between η and nucleon which is the following functional form,

$$V_{\eta N}(r) = a_1 \exp(-b_1(r_1 - r)^2) - a_2 \exp(-b_2 r^2) - i\, a_3 \exp(-b_3 r^2) \tag{9}$$

with parameters a's, b's and r_1 from "MSc Thesis" (J.S. De Villiers, 2005). The value of their parameters are described in Table (1) and shown in Fig. (1) and Fig(2).

Table 1 Parameters of the η-N potential

Parameter	*Value*
r_1	1.95616478619031975fm
a_1	57.5826586837329657MeV
a_2	26.8157044304329091MeV
a_3	0.603932024464326478MeV
b_1	0.0715471865601824408fm^{-2}
b_2	0.0271505486074286040 fm^{-2}
b_3	0.0338015704618582769 fm^{-2}

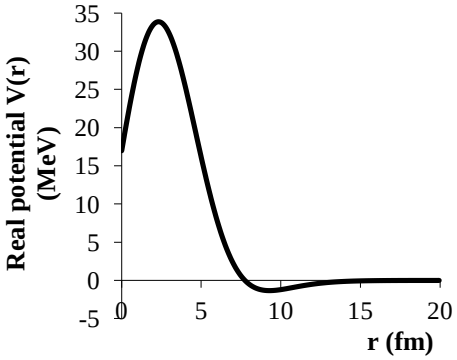

Figure 1 Real part of the Eta-Nucleon Potential

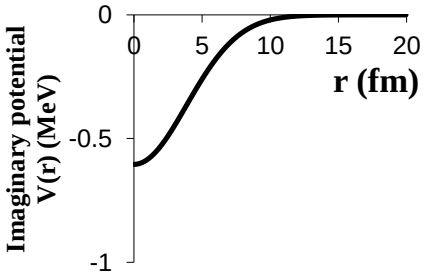

Figure 2 Imaginary part of the Eta-Nucleon Potential

Transformation of Functional form to Gaussian Form

Real part of Gaussian potential between η and nucleon has the following form,

$$V_{\eta\text{-}N}^{\text{Real}}(r) = \sum_{k=1}^{N} V_k \, e^{-\left(\frac{r}{\mu_k}\right)^2} \tag{10}$$

where, V_k and μ_k are the potential strengths and range parameters which are adjusted in the calculations with

$$\mu_N = c^{N-1}\mu_1 \, , \, c = \left(\frac{\mu_N}{\mu_1}\right)^{\frac{1}{N-1}} \tag{11}$$

The transformation equation can be expressed as

$$V_{\eta\text{-}N}^{\text{Real}}(r) = \sum_{k=1}^{N} V_k \, e^{-\left(\frac{r}{\mu_k}\right)^2} . \tag{12}$$

Multiplying both sides of the equation by $e^{-\left(\frac{r}{\mu_\ell}\right)^2}$ from the right and integration through the equation, we obtain

$$\int_0^{\infty} V_{\eta\text{-}N}^{\text{Real}}(r)\; e^{-\left(\frac{r}{\mu_\ell}\right)^2}\; dr = \sum_{k=1}^{N} \int_0^{\infty} V_k \;\; e^{-\left(\frac{r}{\mu_k}\right)^2} \; e^{-\left(\frac{r}{\mu_\ell}\right)^2} \;\; dr. \qquad (13)$$

The left hand side of the equation (13) was numerically solved and the right hand side was solved by using standard integral form.

The above equation becomes

$$\int_0^{\infty} V_{\eta\text{-}N}^{\text{Real}}(r) e^{-\left(\frac{r}{\mu_\ell}\right)^2} \; dr = \frac{1}{2} \sum_{k=1}^{N} V_k \sqrt{\frac{\pi}{\frac{1}{\mu_k^2}+\frac{1}{\mu_\ell^2}}} \; . \qquad (14)$$

$$\text{Let } A_\ell = \int_0^{\infty} V_{\eta\text{-}N}^{\text{Real}}(r) \; e^{-\left(\frac{r}{\mu_\ell}\right)^2} \; dr \text{ and } B_{\ell k} = \frac{1}{2} \sqrt{\frac{\pi}{\frac{1}{\mu_k^2}+\frac{1}{\mu_\ell^2}}} \qquad (15)$$

The equation (14) is written as

$$A_\ell = \sum_{k=1}^{N} B_{\ell k} \; V_k$$

where ℓ goes from 1 to N

$$A_1 = B_{11} V_1 + B_{12} V_2 + B_{13} V_3 + \cdots + B_{1N} V_N$$
$$A_2 = B_{21} V_1 + B_{22} V_2 + B_{23} V_3 + \cdots + B_{2N} V_N$$
$$\vdots$$
$$A_N = B_{N1} V_1 + B_{N2} V_2 + B_{N3} V_3 + \cdots + B_{NN} V_N$$

These N linear equations are solved by using the Gauss elimination Method. The transformed Gaussian potential is found to be $\mu_1 = 0.05$ fm, $\mu_N = 20$ fm and N=20 which are shown in Table (2).

Table 2 The range parameters and potential strengths of the transformed η-N Gaussian potential

Range parameter (fm)	***Potential strength(MeV)***
μ(1) = 0.10000	V(1) = -0.65741
μ (2) = 0.13216	V(2) = 0.89916
μ (3) = 0.17466	V(3) = -3.57331
μ (4) = 0.23084	V(4) = 6.28714
μ (5) = 0.30508	V(5) = -11.5881
μ (6) = 0.40321	V(6) = 14.70480
μ (7) = 0.53289	V(7) = -21.12319
μ (8) = 0.70428	V(8) = 23.41650
μ (9) = 0.93079	V(9) = -32.04554
μ (10) = 1.23015	V(10) = 33.71635
μ (11) = 1.62580	V(11) = -49.02017
μ (12) = 2.14869	V(12) = 52.08892
μ (13) = 2.83976	V(13) = -101.83089
μ (14) = 3.75308	V(14) = 59.00174
μ (15) = 4.96016	V(15) = 86.98296
μ (16) = 6.55545	V(16) = -49.78051
μ (17) = 8.66382	V(17) = 13.37485
μ (18) = 11.45028	V(18) = -4.59364
μ (20) = 20.00000	V(20) = -0.21711

Eta-Nucleus Interaction

The interaction between η and the core nucleus ^{16}O is obtained by folding the $V_{\eta N}$ interaction with the density distribution of ^{16}O as

$$V_{\eta-\text{nucl}}(\vec{R}) = \int V_{\eta N}(\vec{R}-\vec{r})\,\rho_{\text{nucl}}(\vec{r})\,d\vec{r} \qquad (16)$$

where $\vec{R}$ is the distance betweenη particle and the center of the mass of the core nucleus and $\rho_{\text{nucl}}(\vec{r})$ is the nuclear density distribution of the core nucleus ^{16}O, $\vec{r}$ is the distance between nucleon and center of mass of the core nucleus as shown in Fig. (3). $V_{\eta-\text{nucl}}(\vec{R})$ is the folding potential of η and the core nucleus ^{16}O system. The density distribution of $\rho_{\text{nucl}}(\vec{r})$ in harmonic oscillator model is given as follows;

$$\rho_{\text{nucl}}(\vec{r}) = \rho_0\left\{1+\frac{\alpha}{a^2}r^2\right\}e^{-\left(\frac{r}{a}\right)^2} \qquad (17)$$

where,α=1.544, a=1.833fm from [8].

$$\rho_0 = \frac{A}{a^3\pi^{3/2}\,(1+1.5\alpha)}$$

where, A=mass number of the core nucleus.

For $^{16}_{8}O$ nucleus, the density of nuclear matter ρ_0=0.14067 fm^{-3}.

Then the folding potential for the phenomenological η-N interaction is

$$V_{\eta-nucl}(\vec{R}) = \int \rho_{nucl}(\vec{r})\, V_{\eta N}(\vec{R}-\vec{r})d\vec{r}$$

$$V_{\eta-nucl}(\vec{R}) = \int \left\{\rho_0 + Y\, r^2\right\} e^{-\left(\frac{r}{a}\right)^2} \sum_{K=1}^{N} V_K\, e^{-\left(\frac{\vec{r}-\vec{R}}{\mu_K}\right)^2} d\vec{r} \qquad (18)$$

$$V_{\eta-nucl}(\vec{R}) = \sum_{K=1}^{N} V_K \left(\frac{\pi}{A}\right)^{3/2} \left\{ P\, e^{-\left(\frac{\vec{R}}{D_K}\right)^2} + Q\, R^2\, e^{-\left(\frac{\vec{R}}{D_K}\right)^2} \right\}$$

where, $P = \rho_0 + \frac{1.5}{B}Y$, $Q = \frac{Y}{B^2\mu_K^4}$, $Y = \rho_0 \frac{\alpha}{a^2} = 0.0646$,

$$B = \frac{1}{a^2} + \frac{1}{\mu_K^2} \text{ and } \frac{1}{D_K^2} = \frac{1}{\mu_K^2} - \frac{1}{B\mu_K^4}.$$

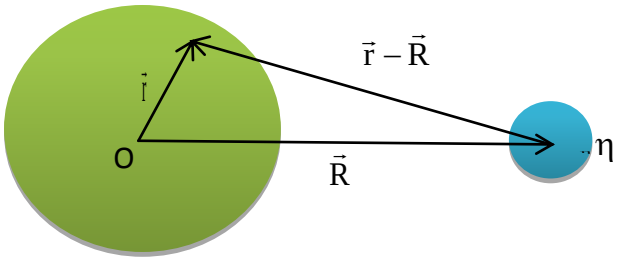

Figure 3 The core nucleus and η particle

Density of nuclear matter (ρ_0)

We find the existence of the value ρ_0, known as the 'density of nuclear matter'. The normalization of the charge distribution is

$$4\pi\int \rho(r) r^2 dr = Ze. \qquad (19)$$

The density is only approximate, since we have neglect the finite size of both proton and neutron and the effect of Coulomb forces, but it indicates that at the centre of a nucleus.

$$\int \rho_{nucl}(\vec{r})\, d\vec{r} = A \qquad (20)$$

where A=mass number of the core nucleus

$$4\pi\int \rho(r) r^2 dr = A$$

The Harmonic Oscillator density distribution is Gaussian

$$\rho(r) = \rho_0\left[1+\alpha\left(\frac{r}{a}\right)^2\right]e^{-\left(\frac{r}{a}\right)^2} \quad .$$

Equation (19) becomes

$$4\pi\int \rho_0\left[1+\alpha\left(\frac{r}{a}\right)^2\right]e^{-\left(\frac{r}{a}\right)^2} r^2 dr = Ze \qquad (21)$$

By using the Standard Integral form $\int_0^\infty r^{2N}e^{-Ar^2}dr = \frac{(2N-1)!!}{2^{n+1}}\frac{1}{A^N}\sqrt{\frac{\pi}{A}}$ and

the result is $$\rho_0\left[\frac{1!!}{2^2}\frac{1}{1/a^2}\sqrt{\frac{\pi}{1/a^2}}+\frac{\alpha}{a^2}\frac{3!!}{2^3}\frac{1}{1/a^4}\sqrt{\frac{\pi}{1/a^2}}\right]=\frac{Ze}{4\pi}$$

$$\rho_0 = \frac{Ze}{4\pi}\frac{4}{a^3\sqrt{\pi}\left[1+3/2\alpha\right]}$$

For $^{16}_{8}O$ nucleus, Ze= 16.

Substituting the parameter a=1.833fm, $\alpha = 1.544$, π=3.142 and A=16 for $^{16}_{8}O$,we get ρ_0 is 0.14067fm^{-3}.

Results and Discussion

Binding Energy and Level Width of $^{16}_{\eta}O$ Nucleus

Our research is to investigate the structure of η-mesic Oxygen nucleus $^{16}_{\eta}O$ theoretically, considering as a bound system of η-meson and Oxygen core nucleus. It is found that although the η-N interaction is attractive, it is not strong enough to have a two-body bound state. Since Oxygen is the double magic nucleus, it is a very high stability nucleus. Therefore we expect to be a bound system of double magic oxygen nucleus and η-meson.

Before performing the studying of η-meson Oxygen system, we have transformed the functional form into Gaussian form which is nearly equivalent to original potential by applying the Gauss elimination method. The η-N interaction which is functional form cannot be analytically solved. The transformed Gaussian potential is obtained by applying the optimum parameter set, $\mu_1 = 0.05$ fm, $\mu_N = 20$ fm and N=20. These transformed potential compared with original potential is shown in Fig.(4).

To study the η-nucleus bound system, we have to know the interaction between η and oxygen core nucleus. By folding the η-N interaction with oxygen nucleus nuclear density, we obtained the η-nucleus interaction. And then we have computed the eigen value of $^{16}_{\eta}O$ system with the η-nucleus folding potential. Oxygen nucleus nuclear density for Harmonic Oscillator

model is $\rho_0 = \frac{A}{a^3\pi^{3/2}\left[1+1.5\,\alpha\right]}$. By applying the parameter a=1.833fm, $\alpha = 1.544$,π=3.142 and A=16, our calculated nuclear density ρ_0 is 0.14067 fm^{-3}.

In order to calculate the structure of two-body quantum system we solved Schrödinger equation. To solve the Schrödinger radial equation we use the Gaussian basis wave function. We have calculated the eigenvalue E with corresponding eigenvectors C_j by using FORTRAN CODE. To solve the eigen value equation, we used the Power Inverse Iteration Method. Our calculated result is 3.2814 MeV and its level width is 0.3704 MeV.

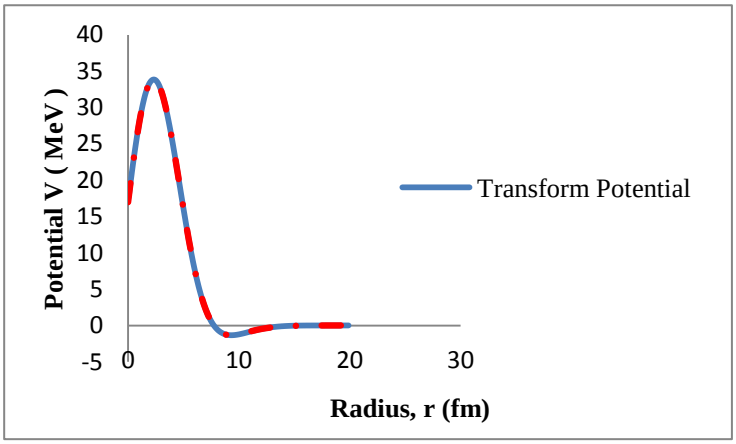

Figure 4 The real part of potentials between η and the nucleon, solid and dotted curve indicate the functional form and the transformed Gaussian potential, respectively.

Conclusion

Our thesis is to investigate the structure of η-mesic Oxygen nucleus ${}^{16}_{\eta}O$ theoretically, considering as a bound system of η-meson and Oxygen core nucleus. To study the η-Oxygen nucleus bound system, we have to know the interaction between η and Oxygen core nucleus ${}^{16}_{8}O$. The η-N interaction which is functional form cannot be analytically solved, so we transformed the functional form into Gaussian form by applying the Gauss elimination method. By folding η-N interaction with Oxygen core nucleus nuclear density we obtained the η-nucleus interaction. And then we compute the eigen value of ${}^{16}_{\eta}O$ system with the η-nucleus folding potential. Our calculated result is 3.2814 MeV with a level width is 0.3704 MeV.

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